Appendix X to:
A Theoretical Investigation to the Physical Constraints for Light Velocity in Empty Euclidian Space and first Consequences for Long-distance Physics.
Referred equations: (34a), (35), (36), (37), (B.19), (B.20)

Space transformations for force-free moving systems.

A coordinate transformation is handled for a force-free moving system, for which the time transformation was proven to be:

(35) $T^*(T_{or},d_\infty;x,y,T) = \Theta_{or}(\gamma^2 - x\,d_\infty)\,T - \Delta t_{or}$

and considering the vectors $\vec{s}$ and $\vec{r}$, photon direction μ and transformation functions $X(\,)$ and $Y(\,)$ in polar coordinates as in Appendix B. The above time transformation is inserted into equation (B.19) where $x = s\cos(\sigma)$:

(X.1) $$Y(s,\sigma,T_s) = C_d\, s\sin(\sigma)\frac{T_s}{T_s^*} = \frac{C_d\, s\sin(\sigma)\, T_s}{\Theta_{or}\{\gamma^2 - s\,d_\infty\cos(\sigma)\}\,T_s - \Delta t_{or}}$$

At this point, we can repeat the treatment of the non-moving case; in the limit for $T_s \rightarrow \infty$, $s \rightarrow \gamma$, $\sigma \rightarrow \mu$ the above equation becomes:

(X.2) $$Y(\gamma,\mu,\infty) = \frac{C_d\sin(\mu)}{\Theta_{or}\{\gamma - d_\infty\cos(\mu)\}}$$

and that value will have a maximum of γ reached for a μ_0 for which $dY/d\mu = 0$

(X.3) $$0 = \frac{\cos(\mu_0)}{\gamma - d_\infty\cos(\mu_0)} - \frac{d_\infty\sin(\mu_0)^2}{\{\gamma - d_\infty\cos(\mu_0)\}^2}$$

where the constants C_d, Θ_{or} have been left out; the solution for μ_0 is:

(X.4) $$\cos(\mu_0) = \frac{d_\infty}{\gamma} \qquad \Rightarrow \qquad \sin(\mu_0) = \sqrt{1 - \frac{d_\infty^2}{\gamma^2}}$$

Then, equation (X.2) gives:

(X.5) $$Y(\gamma,\mu_0,\infty) = \gamma = \frac{C_d\sin(\mu_0)}{\Theta_{or}\{\gamma - d_\infty\cos(\mu_0)\}} = \frac{C_d\sqrt{1 - d_\infty^2/\gamma^2}}{\Theta_{or}(\gamma - d_\infty^2/\gamma)}$$

and C_d can be expressed in terms of Θ_{or}:

(X.6) $$C_d = \Theta_{or}\,\gamma\sqrt{\gamma^2 - d_\infty^2}$$

Now, the general solution for $Y(\,)$ becomes – from equation (X.1) now written in terms of r, α, T_r:

(X.7) $$Y(r,\alpha,T_r) = \frac{\gamma\sqrt{\gamma^2 - d_\infty^2}\; r\sin(\alpha)\, T_r}{\{\gamma^2 - r\,d_\infty\cos(\alpha)\}\,T_r - \Delta t_{or}/\Theta_{or}}$$

In the limit for $T_r \rightarrow \infty$, $r \rightarrow \gamma$, $\alpha \rightarrow \mu$:

(X.8) $$Y(\gamma,\mu,\infty) = \frac{\gamma\sqrt{\gamma^2 - d_\infty^2}\,\sin(\mu)}{\gamma - d_\infty\cos(\mu)}$$

and, because $X(\gamma,\mu,\infty)^2 + Y(\gamma,\mu,\infty)^2$ has to be equal to γ^2:

(X.9) $$X(\gamma,\mu,\infty) = \sqrt{\gamma^2 - \frac{\gamma^2(\gamma^2 - d_\infty^2)\sin(\mu)^2}{\{\gamma - d_\infty\cos(\mu)\}^2}} = \frac{\gamma\{\gamma\cos(\mu) - d_\infty\}}{\gamma - d_\infty\cos(\mu)}$$

where the + sign of the square root pertains since $X(\,) > 0$ for $\mu = 0$. Generally, for the x-components, it was shown in Appendix B that:

(B.20) $$X(r,\alpha,T_r)\,T_r^* = X(\gamma,\mu,\infty)\;(T_r^* - T_s^*) + X(s,\sigma,T_s)\,T_s^*$$

and again, we will take the special case of $\vec{s}$ being the origin of the target system at time T_d, $T_s = T_d$, $\vec{s} = \vec{d}_\infty(1 - T_{or}/T_d)$ so that $X(s(T_d),0,T_d) = 0$. Applying equation (34a) to equation (35):

(X.10) $$T_s^* = \Theta_{or}\{(\gamma^2 - d_\infty^2)\,T_d + d_\infty^2\,T_{or}\} - \Delta t_{or}$$

then applying equations (35), (X.9), (X.10) to equation the right part of (B.20):

(X.11) $$X(r,\alpha,T_r)\,T_r^* = \frac{\gamma\{\gamma\cos(\mu) - d_\infty\}}{\gamma - d_\infty\cos(\mu)}\Big(\Theta_{or}\{\gamma^2 - r\,d_\infty\cos(\alpha)\}\,T_r - \Delta t_{or} - \Theta_{or}[\{(\gamma^2 - d_\infty^2)\,T_d + d_\infty^2\,T_{or}\} - \Delta t_{or}]\Big)$$

Again T_r and T_d are connected by a photon path, now starting from a time-dependent position:

(X.12) $$(\vec{\gamma} - \vec{r})\,T_r = \{\vec{\gamma} - \vec{d}(T_d)\}\,T_d = (\vec{\gamma} - \vec{d}_\infty)\,T_d + \vec{d}_\infty\,T_{or}$$

Taking the x- components of this path equation gives:

(X.13) $$\{\gamma\cos(\mu) - r\cos(\alpha)\}\,T_r = \{\gamma\cos(\mu) - d_\infty\}\,T_d + d_\infty\,T_{or}$$

and inserting the resulting T_d in the above equation for $X(r,\alpha,T_r)\,T_r^*$:

(X.14) $$X(r,\alpha,T_r)\,T_r^* = \frac{\gamma\Theta_{or}}{\gamma - d_\infty\cos(\mu)}\Big(\{\,\gamma\cos(\mu) - d_\infty\,\}\,[\,\{\,\gamma^2 - r\,d_\infty\cos(\alpha)\,\}\,T_r - d_\infty{}^2\,T_{or}\,]$$
$$- (\gamma^2 - d_\infty{}^2)\,[\,\{\,\gamma\cos(\mu) - r\cos(\alpha)\,\}\,T_r - d_\infty\,T_{or}\,]\Big)$$

This can be simplified to:

(X.15) $$X(r,\alpha,T_r)\,T_r^* = \gamma^2\,\Theta_{or}\,[\,\{\,r\cos(\alpha) - d_\infty\,\}\,T_r + d_\infty\,T_{or}\,]$$

so that, also expliciting T_r^* and dropping the subscript r :

(X.16) $$X(r,\alpha,T) = \frac{\gamma^2\,[\,\{\,r\cos(\alpha) - d_\infty\,\}\,T + d_\infty\,T_{or}\,]}{\{\,\gamma^2 - r\,d_\infty\cos(\alpha)\,\}\,T - \Delta t_{or}/\Theta_{or}}$$

Switching to Cartesian coordinates this is equation (36):

(36) $$x^*(T_{or},d_\infty;x,y,T) = \gamma^2\,\frac{(x - d_\infty)\,T + d_\infty\,T_{or}}{(\gamma^2 - x\,d_\infty)\,T - \Delta t_{or}/\Theta_{or}}$$

where equation (X.7) becomes equation (37):

(37) $$y^*(T_{or},d_\infty;x,y,T) = \frac{\gamma\sqrt{\gamma^2 - d_\infty{}^2}\;y\,T}{(\gamma^2 - x\,d_\infty)\,T - \Delta t_{or}/\Theta_{or}}$$
