Appendix R to:
A Theoretical Investigation to the Physical Constraints for Light Velocity in Empty Euclidian Space and first Consequences for Long-distance Physics.
Referred equations: (9), (13), (16)

*Derivation of the travel time relation equation for displacement*

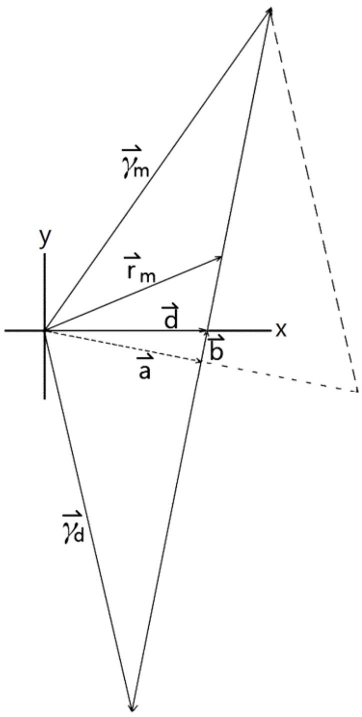
Considering the two photon trajectories of equation (13), from $\vec{d}$ at time $T_1$ to $\vec{r}_m$ at time $T_m$ and back from $\vec{r}_m$ at time $T_m$ to $\vec{d}$ at time $T_2$, and inserting these into the equation for the photon path, equation (9), results in:

(R.1) $(\vec{\gamma}_m - \vec{r}_m)\, T_m = (\vec{\gamma}_m - \vec{d})\, T_1$
$(\vec{\gamma}_d - \vec{r}_m)\, T_m = (\vec{\gamma}_d - \vec{d})\, T_2$

where $\vec{\gamma}_m$ and $\vec{\gamma}_d$ are the respective limit locations of the photons. Both equations can be combined into an equation for the summed times $T_1 + T_2$:

(R.2) $(\vec{\gamma}_m - \vec{d})\bullet(\vec{\gamma}_d - \vec{d})\,(T_1 + T_2) = \{\, (\vec{\gamma}_m - \vec{r}_m)\bullet(\vec{\gamma}_d - \vec{d}) + (\vec{\gamma}_m - \vec{d})\bullet(\vec{\gamma}_d - \vec{r}_m) \,\}\, T_m$

where • denotes the vector inner product. The situation of equation (R.1) is schematically given in the figure at the right. $\vec{\gamma}_d$, $\vec{d}$, $\vec{r}_m$ and $\vec{\gamma}_m$ are on the same trajectory, the part between $\vec{d}$ and $\vec{r}_m$ travelled forth and back respectively by either photon. Perpendicular to that, a construction line $\vec{a}$ is drawn: $\vec{a} = ½(\vec{\gamma}_d + \vec{\gamma}_m)$, and the distance to $\vec{d}$ is indicated as $\vec{b}$: $\vec{b} = \vec{d} - \vec{a}$. Since the triangle formed by $\vec{\gamma}_d$, $\vec{\gamma}_m$ is isosceles (both these sidelines have a length $\gamma$) $\vec{\gamma}_d - \vec{a} = \vec{a} - \vec{\gamma}_m$ where their length is $\sqrt{\gamma^2 - a^2}$. Then, for the left-hand part of equation (R.2) – note, that $\vec{d} = \vec{a} + \vec{b}$:




(R.3) $(\vec{\gamma}_m - \vec{d})\bullet(\vec{\gamma}_d - \vec{d}) = (\vec{\gamma}_m - \vec{a} - \vec{b})\bullet(\vec{\gamma}_d - \vec{a} - \vec{b})$
$= (\vec{\gamma}_m - \vec{a})\bullet(\vec{\gamma}_d - \vec{a}) - (\vec{\gamma}_m + \vec{\gamma}_d - 2\,\vec{a})\bullet\vec{b} + b^2$
$= -(\gamma^2 - a^2) + 0 + b^2$
$= -(\gamma^2 - d^2)$

And for the right-hand part of equation (R.2):

(R.4) $(\vec{\gamma}_m - \vec{d})\bullet(\vec{\gamma}_d - \vec{r}_m) + (\vec{\gamma}_m - \vec{r}_m)\bullet(\vec{\gamma}_d - \vec{d})$
$= (\vec{\gamma}_m - \vec{d})\bullet\{\, \vec{\gamma}_d - \vec{d} + (\vec{d} - \vec{r}_m) \,\} + \{\, \vec{\gamma}_m - \vec{d} + (\vec{d} - \vec{r}_m) \,\}\bullet(\vec{\gamma}_d - \vec{d})$
$= 2\,(\vec{\gamma}_m - \vec{d})\bullet(\vec{\gamma}_d - \vec{d}) + (\vec{\gamma}_m + \vec{\gamma}_d)\bullet(\vec{d} - \vec{r}_m) - 2\,\vec{d}\bullet(\vec{d} - \vec{r}_m)$

However, $\vec{\gamma}_m + \vec{\gamma}_d = 2\,\vec{a}$ is perpendicular to $\vec{d} - \vec{r}_m$ so that their inner product is zero. Then, also using the first result, we derive:

(R.5) $(\vec{\gamma}_m - \vec{d})\bullet(\vec{\gamma}_d - \vec{r}_m) + (\vec{\gamma}_m - \vec{r}_m)\bullet(\vec{\gamma}_d - \vec{d})$
$= -2\,(\gamma^2 - d^2) + 0 - 2\,d^2 + 2\,\vec{d}\bullet\vec{r}_m$
$= -2\,(\gamma^2 - \vec{d}\bullet\vec{r}_m)$
$= -2\,(\gamma^2 - x_m\, d)$

so that equation (R.2) becomes:

(16) $(\gamma^2 - d^2)\,(T_1 + T_2) = 2\,(\gamma^2 - x_m\, d)\, T_m$

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