Appendix F to:
A Theoretical Investigation to the Physical Constraints for Light Velocity in Empty Euclidian Space and first Consequences for Long-distance Physics.
Referred equations: (9), (14), (27), (28)

Derivation of the basic equation for Θ( ) for moving systems transformations

A system is considered moving in the x-direction:

(F.1) $$\vec{d} = \begin{pmatrix} d(T) \\ 0 \\ 0 \end{pmatrix}$$

and a photon travelling from the system's origin at time T_1 forward to a location x_m also on the x-axis and reached at time T_x then mirrored back towards the system's origin reached at time T_2:

(F.2) $$\begin{aligned} x_{ph}(T_1) &= d(T_1) \qquad & x^*_{ph}(T^*_1) &= 0 \\ x_{ph}(T_x) &= x_m \qquad & x^*_{ph}(T^*_x) &= x^*_m \\ x_{ph}(T_2) &= d(T_2) \qquad & x^*_{ph}(T^*_2) &= 0 \end{aligned}$$

The photon's path is described by equation (9):

(F.3) $$\begin{aligned} x\,T &= d(T_1)\,T_1 + \gamma\,(T - T_1) \qquad & T \le T_x \\ x\,T &= d(T_2)\,T_2 - \gamma\,(T - T_2) \qquad & T \ge T_x \end{aligned}$$

where x is the photon's location at time T. Again, for the transformed system equation (14) is valid:

(14) $$2T^*_x = T^*_1 + T^*_2$$

and hence, from equation (27):

(F.4) $$2\,\Theta(T_{or},v_{or};x_m,0)\,T_x = \Theta(T_{or},v_{or};d(T_1),0)\,T_1 + \Theta(T_{or},v_{or};d(T_2),0)\,T_2$$

Now, we consider $T = T_x$ in equation (F.3) and make substitutions $T_1 = T_x - \delta T_1$, $T_2 = T_x + \delta T_2$, $v(T) = \frac{d}{dT} d(T)$ where δT_1, δT_2 are infinitesimally small:

(F.5) $$\begin{aligned} x_m\,T_x &= d(T_x)\,T_x + \{\,\gamma - d(T_x) - v(T_x)\,T_x\,\}\,\delta T_1 \\ x_m\,T_x &= d(T_x)\,T_x + \{\,\gamma + d(T_x) + v(T_x)\,T_x\,\}\,\delta T_2 \end{aligned}$$

so that, defining

(F.6) $$\varepsilon = x_m - d(T_x)$$

we can write δT_1, δT_2 as:

(F.7) $$\delta T_1 = \frac{\varepsilon\,T_x}{\gamma - d(T_x) - v(T_x)\,T_x} \qquad\qquad \delta T_2 = \frac{\varepsilon\,T_x}{\gamma + d(T_x) + v(T_x)\,T_x}$$

Then, equation (F.4) can be worked out as – leaving out the parameters T_{or}, v_{or} for better readability:

(F.8) $$\begin{aligned} 2\left\{\Theta(d(T_x),0) + \varepsilon\frac{\partial\Theta}{\partial x}(d(T_x),0)\right\}T_x &= \Theta(d(T_x),0)\,T_x - \left\{\Theta(d(T_x),0) + v(T_x)\,T_x\frac{\partial\Theta}{\partial x}(d(T_x),0)\right\}\frac{\varepsilon\,T_x}{\gamma - d(T_x) - v(T_x)\,T_x} \\ &+ \Theta(d(T_x),0)\,T_x + \left\{\Theta(d(T_x),0) + v(T_x)\,T_x\frac{\partial\Theta}{\partial x}(d(T_x),0)\right\}\frac{\varepsilon\,T_x}{\gamma + d(T_x) + v(T_x)\,T_x} \end{aligned}$$

which after some rearrangement yields:

(F.9) $$\frac{\partial\Theta}{\partial x}(d(T_x),0)\,[\,\gamma^2 - d(T_x)\,\{\,d(T_x) + v(T_x)\,T_x\,\}\,] = -\,\{\,d(T_x) + v(T_x)\,T_x\,\}\,\Theta(d(T_x),0)$$

which in fact is a differential equation in $\Theta(\,)$ as soon as the relation between $d(\,)$ and T_x is established. Equation (28) is the above equation where the parameters T_{or}, v_{or} are shown also.

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