Appendix C to:
A Theoretical Investigation to the Physical Constraints for Light Velocity in Empty Euclidian Space and first Consequences for Long-distance Physics.

Referred equations: (4), (5)

Derivation of the basic equations for light velocity.

Starting point of these derivations is the equality:

(4) $\Psi(\vec{r},t+\Delta t) \equiv \Psi(\vec{r}-\vec{c}(\vec{r},t)\,\Delta t,t)$

The n^{th} derivative to Δt is:

(C.1) $$\frac{\partial^n\Psi}{\partial t^n}(\vec{r},t+\Delta t) \equiv (-\vec{c}(\vec{r},t)\bullet\underline{\nabla}\underline{)^n\,\Psi}(\vec{r}-\vec{c}(\vec{r},t)\,\Delta t,t)$$

where, as indicated, the operator ∇ operates on Ψ only and not on $\vec{c}$. In the limit for $\Delta t \to 0$:

(C.2) $$\frac{\partial^n\Psi}{\partial t^n} \equiv (-\vec{c}\bullet\underline{\nabla}\underline{)^n\,\Psi}$$

where both $\vec{c}$ and Ψ are dependent on $(\vec{r},t)$. For reasons of readability, this is from here on no longer written explicitly.

The second derivative to t also can be derived by differentiating the $n=1$ case of equation (C.2) to t:

(C.3) $$\frac{\partial^2\Psi}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial\Psi}{\partial t}\right) = \frac{\partial}{\partial t}\left(-\vec{c}\bullet\underline{\nabla}\,\underline{\Psi}\right) = -\frac{\partial\vec{c}}{\partial t}\bullet\underline{\nabla}\,\underline{\Psi} - \vec{c}\bullet\underline{\nabla}\,\underline{\left(\frac{\partial\Psi}{\partial t}\right)} = -\frac{\partial\vec{c}}{\partial t}\bullet\underline{\nabla}\,\underline{\Psi} - \vec{c}\bullet\underline{\nabla}\,\underline{\left(-\vec{c}\bullet\nabla\,\Psi\right)}$$
$$= -\frac{\partial\vec{c}}{\partial t}\bullet\underline{\nabla}\,\underline{\Psi} + (\vec{c}\bullet\underline{\nabla})\,\underline{\vec{c}}\bullet\underline{\nabla}\,\underline{\Psi} + (\vec{c}\bullet\underline{\nabla}\underline{)^2\,\Psi}$$

Combination with the $n=2$ case of equation (C.2) shows that:

(C.4) $$\frac{\partial\vec{c}}{\partial t} = (\vec{c}\bullet\underline{\nabla})\,\underline{\vec{c}}$$

which is the top of equation (5). Similarly, for the third derivative:

(C.5) $$\frac{\partial^3\Psi}{\partial t^3} = \frac{\partial}{\partial t}\left(\frac{\partial^2\Psi}{\partial t^2}\right) = \frac{\partial}{\partial t}\left((\vec{c}\bullet\underline{\nabla}\underline{)^2\,\Psi}\right) = 2\left(\frac{\partial\vec{c}}{\partial t}\bullet\underline{\nabla}\right)\underline{(\vec{c}\bullet\nabla)\,\Psi} + (\vec{c}\bullet\underline{\nabla}\underline{)^2\left(\frac{\partial\Psi}{\partial t}\right)} = 2\left(\frac{\partial\vec{c}}{\partial t}\bullet\underline{\nabla}\right)\underline{(\vec{c}\bullet\nabla)\,\Psi} + (\vec{c}\bullet\underline{\nabla}\underline{)^2\left(-\vec{c}\bullet\nabla\,\Psi\right)}$$
$$= 2\left(\frac{\partial\vec{c}}{\partial t}\bullet\underline{\nabla}\right)\underline{(\vec{c}\bullet\nabla)\,\Psi} - (\vec{c}\bullet\underline{\nabla}\underline{)^2\,\vec{c}}\bullet\underline{\nabla}\,\underline{\Psi} - 2\left\{\left((\vec{c}\bullet\underline{\nabla})\,\underline{\vec{c}}\right)\bullet\underline{\nabla}\right\}\underline{(\vec{c}\bullet\nabla)\,\Psi} - (\vec{c}\bullet\underline{\nabla}\underline{)^3\,\Psi}$$

so that, according to equation (C.4):

(C.6) $$\frac{\partial^3\Psi}{\partial t^3} = -(\vec{c}\bullet\underline{\nabla}\underline{)^2\,\vec{c}}\bullet\underline{\nabla}\,\underline{\Psi} - (\vec{c}\bullet\underline{\nabla}\underline{)^3\,\Psi}$$

Combination with the $n=3$ case of equation (C.2) shows that:

(C.7) $$(\vec{c}\bullet\underline{\nabla}\underline{)^2\,\vec{c}} = 0$$

which is the bottom of equation (5).

It still has to be shown, that the different ways of deriving $\partial^n\Psi/\partial t^n$ are equivalent also for $n>3$. Assuming that to be true up to and including a given value of n, we derive for the $n+1$ case, keeping in mind equation (C.7):

(C.8) $$\frac{\partial^{n+1}\Psi}{\partial t^{n+1}} = \frac{\partial}{\partial t}\left(\frac{\partial^n\Psi}{\partial t^n}\right) = \frac{\partial}{\partial t}\left((-\vec{c}\bullet\underline{\nabla}\underline{)^n\,\Psi}\right) = -n\left(\frac{\partial\vec{c}}{\partial t}\bullet\underline{\nabla}\right)\underline{(-\vec{c}\bullet\nabla)^{n-1}\,\Psi} + (-\vec{c}\bullet\underline{\nabla}\underline{)^n\left(\frac{\partial\Psi}{\partial t}\right)}$$
$$= -n\left(\frac{\partial\vec{c}}{\partial t}\bullet\underline{\nabla}\right)\underline{(-\vec{c}\bullet\nabla)^{n-1}\,\Psi} + (-\vec{c}\bullet\underline{\nabla}\underline{)^n\left(-\vec{c}\bullet\nabla\,\Psi\right)}$$
$$= -n\left(\frac{\partial\vec{c}}{\partial t}\bullet\underline{\nabla}\right)\underline{(-\vec{c}\bullet\nabla)^{n-1}\,\Psi} + n\left\{\left((\vec{c}\bullet\underline{\nabla})\,\underline{\vec{c}}\right)\bullet\underline{\nabla}\right\}\underline{(-\vec{c}\bullet\nabla\,)^{n-1}\,\Psi} + (-\vec{c}\bullet\underline{\nabla}\underline{)^{n+1}\,\Psi}$$

Again using equation (C.4), this indeed verifies the $n+1$ case of equation (C.2) so that it is true for any value of n. Consequently, the representation of equation (4) is mathematically possible.
