Appendix B to:
A Theoretical Investigation to the Physical Constraints for Light Velocity in Empty Euclidian Space and first Consequences for Long-distance Physics.
Referred equations: (9),(12)

Derivation of the basic time transform equation

A transformation is considered along the x-direction. For a vector $\vec{s}$ located in the x,y plane:

(B.1) $$\vec{s} = \begin{pmatrix} s_x \\ s_y \\ 0 \end{pmatrix} = \begin{pmatrix} s\cos(\sigma) \\ s\sin(\sigma) \\ 0 \end{pmatrix}$$

where we will use the rightmost notation, in polar coordinates, because that makes the treatment easier. It is assumed that in principle there is no preferred space direction. Then, the transformation into the new coordinate system at (absolute) time T_s can be written as:

(B.2) $$\vec{s}^* = \begin{pmatrix} s_x^* \\ s_y^* \\ 0 \end{pmatrix} = \begin{pmatrix} X(s,\sigma,T_s) \\ Y(s,\sigma,T_s) \\ 0 \end{pmatrix}$$

where $\vec{s}^*$ is the transformed coordinate, also in the x,y plane. Note, that the functional relationships X( ) and Y( ) also will depend on the describing parameter(s) of the transformation; these are omitted from the notation but can be added later in specific situations.

A vector not in the x,y plane will be in a plane rotated over an angle β around the x axis. That plane should have the same properties as the x,y plane, and consequently, the vector will be transformed also according to the functions X( ) and Y( ) now in the rotated plane. So, for any vector $\vec{r}$ at time T_r:

(B.3) $$\vec{r} = \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix} = \begin{pmatrix} r\cos(\alpha) \\ r\sin(\alpha)\cos(\beta) \\ r\sin(\alpha)\sin(\beta) \end{pmatrix}$$

the transformation will be:

(B.4) $$\vec{r}^* = \begin{pmatrix} r^*_x \\ r^*_y \\ r^*_z \end{pmatrix} = \begin{pmatrix} X(r,\alpha,T_r) \\ Y(r,\alpha,T_r)\cos(\beta) \\ Y(r,\alpha,T_r)\sin(\beta) \end{pmatrix}$$

The vectors $\vec{s}$ and $\vec{r}$ will be connected by a photon path[*], into a direction determined by $\vec{\gamma}$:

(B.5) $$\vec{\gamma} = \begin{pmatrix} \gamma\cos(\mu) \\ \gamma\sin(\mu)\cos(\nu) \\ \gamma\sin(\mu)\sin(\nu) \end{pmatrix}$$

that will be approached at infinite time, $T \to \infty$ for positive T_0 and $T \to -\infty$ for negative T_0, in the following equations both indicated by the symbol ∞; so that also:

(B.6) $$\vec{\gamma}^* = \begin{pmatrix} X(\gamma,\mu,\infty) \\ Y(\gamma,\mu,\infty)\cos(\nu) \\ Y(\gamma,\mu,\infty)\sin(\nu) \end{pmatrix}$$

Where applicable, the direction $\vec{\gamma}$ can be discerned for travel from $\vec{s}$ to $\vec{r}$ as $\vec{\gamma}_r$, angle μ_r, or for travel from $\vec{r}$ to $\vec{s}$ as $\vec{\gamma}_s$, angle μ_s. Note, that the respective (absolute) times T_s and T_r will be related through the photon's travel time; according to equation (9) a photon traveling between $(\vec{s}, T_s)$ and $(\vec{r}, T_r)$ obeys:

(B.7) $$\vec{r}\,T_r = \vec{s}\,T_s + \vec{\gamma}\,(T_r - T_s)$$

so that:

(B.8) $$\vec{r}\,T_r = \begin{pmatrix} \gamma\cos(\mu)\,(T_r - T_s) + s\cos(\sigma)\,T_s \\ \gamma\sin(\mu)\cos(\nu)\,(T_r - T_s) + s\sin(\sigma)\,T_s \\ \gamma\sin(\mu)\sin(\nu)\,(T_r - T_s) \end{pmatrix}$$

and, when applying equation (B.7) now in terms of the transformed coordinates:

(B.9) $$\vec{r}^*\,T_r^* = \begin{pmatrix} X(\gamma,\mu,\infty)\,(T_r^* - T_s^*) + X(s,\sigma,T_s)\,T_s^* \\ Y(\gamma,\mu,\infty)\cos(\nu)\,(T_r^* - T_s^*) + Y(s,\sigma,T_s)\,T_s^* \\ Y(\gamma,\mu,\infty)\sin(\nu)\,(T_r^* - T_s^*) \end{pmatrix}$$

[*] Note, that there is always such path, for at least one direction.

where the T^* is the (absolute) time in the transformed system. This will be related to the original time T but may also depend on the space coordinates. However, in the assumed spatial symmetry, the only relevant space coordinates will be the distance to the origin and the offset angle from the system's displacement, s, σ for T_s and r, α for T_r:

(B.10) $T^*_r = T^*(T_r,r,\alpha)$; $T^*_s = T^*(T_s,s,\sigma)$;

For simplicity, the symbols T^*_r and T^*_s will be used until the above functional relationship is needed.

Since the angle β is the same for both transformed and untransformed systems:

(B.11) $$\mathrm{tg}(\beta) = \frac{r_z}{r_y} = \frac{r^*_z}{r^*_y}$$

we derive from combining $r^*_y r_z = r_y r^*_z$ with equations (B.8), (B.9):

(B.12) $$\{ Y(\gamma,\mu,\infty)\cos(\nu)(T^*_r - T^*_s) + Y(s,\sigma,T_s)\, T^*_s \}\, \gamma \sin(\mu)\sin(\nu)(T_r - T_s)$$
$$= \{ \gamma \sin(\mu)\cos(\nu)(T_r - T_s) + s\sin(\sigma)\, T_s \}\, Y(\gamma,\mu,\infty)\sin(\nu)(T^*_r - T^*_s)$$

so that, after simplification:

(B.13) $$Y(s,\sigma,T_s)\, T^*_s\, \gamma \sin(\mu)(T_r - T_s) = s\sin(\sigma)\, T_s\, Y(\gamma,\mu,\infty)(T^*_r - T^*_s)$$

for any choice of s, σ, μ and T_s leading to:

(B.14) $$\frac{Y(s,\sigma,T_s)\, T^*_s}{s\sin(\sigma)\, T_s} = \frac{Y(\gamma,\mu,\infty)}{\gamma \sin(\mu)} \frac{T^*_r - T^*_s}{T_r - T_s}$$

Now, for $|T_r| \to \infty$ so $r \to \gamma$, $\alpha \to \mu$ (note that this is μ_r, for a photon travelling from $\vec{s}$ to $\vec{r}$) keeping all the other variables constant, the right-hand side will depend on μ only whereas the left-hand side is independent of μ. This implies, that both parts have to be equal to a constant, C_d, to be determined later. In particular, the right-hand part:

(B.15) $$T^*_r - T^*_s = C_d \frac{\gamma \sin(\mu)}{Y(\gamma,\mu,\infty)}(T_r - T_s)$$

Inspection of equation (B.7) learns, that all the parameters in the above equation will not change when varying T_r, T_s in such a way that T_r / T_s remains constant. Then, taking the derivatives $T\partial/\partial T$ for $T=T_r$ and $T=T_s$ respectively yields:

(B.16) $$T_r \frac{\partial T^*}{\partial T}(T_r,r,\alpha) - T_s \frac{\partial T^*}{\partial T}(T_s,s,\sigma) = C_d \frac{\gamma \sin(\mu)}{Y(\gamma,\mu,\infty)}(T_r - T_s)$$

and combination of both equations:

(B.17) $$T_r \frac{\partial T^*}{\partial T}(T_r,r,\alpha) - T^*(T_r,r,\alpha) = T_s \frac{\partial T^*}{\partial T}(T_s,s,\sigma) - T^*(T_s,s,\sigma)$$

again, for any parameter choice. However, the choice is limited since the parameters T_r, r, T_s, s are connected through a photon path. Again considering $|T_r| \to \infty$, the left-hand part will depend on μ only where the right-hand part will not, so that either side has to be equal to a constant, Δt again to be determined later. The general solution of this differential equation is (in terms of the left-hand part):

(B.18) $$T^*(T,r,\alpha) = \Theta(r,\alpha)\, T - \Delta t$$

where now $\Theta(\,)$, Δt are a function and a constant to be determined yet. Switching from polar to Cartesian coordinates gives equation (12):

(12) $$T^*(x,y,T) = \Theta(x,y)\, T - \Delta t$$

In addition to this relation, the left part of equation (B.14) also has to be equal to the constant C_d:

(B.19) $$Y(s,\sigma,T_s) = C_d\, s\sin(\sigma) \frac{T_s}{T^*_s}$$

And for the x-components, combining the x-components of equations (B.4), (B.9) yields:

(B.20) $$X(r,\alpha,T_r)\, T^*_r = X(\gamma,\mu,\infty)(T^*_r - T^*_s) + X(s,\sigma,T_s)\, T^*_s$$

Note, that the above treatment is only valid for a system remaining on the x-axis; on the other hand, any time-dependent behavior along the axis is irrelevant.

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